Enumerative Combinatorics, 5 credits

Enumerativ kombinatorik, 5 högskolepoäng

Course code:	FOUK024
Third-cycle subject:	Mathematics/Applied Mathematics
School:	School of Education, Culture and Communication
Valid from:	Autumn term 2024
Established by:	Dean of the School
Decision date:	2024-03-12
Last modified:	--
Level of education:	Third cycle level
Language	Swedish and English
English version	Yes

Course objective

Enumerative combinatorics – the art of counting – is a beautiful subject with many applications. Once considered to be a bunch of seemingly unrelated methods, nowadays the subject has a mature theory, the most fundamental parts of which belong to the toolbox of any research mathematician. In this course, the student will be introduced to this theory and learn to master it well enough to be able to solve related problems that may arise in a research context and to communicate about it with mathematical peers.

Course content

- Basic combinatorial notions, including permutations, graphs, trees, partitions, lattice paths, multinomial coefficients, Stirling numbers, Catalan numbers and Bell numbers.
- "The twelvefold way", that is, how to count equivalence classes of mappings between two finite sets given one of three possible restrictions on the mappings (none, injectivity, surjectivity) and one of four possible notions of equivalence of mappings (equality, equality up to a permutation of the domain set, equality up to a permutation of the target set, equality up to permutations of both sets).
- Sieve methods, also known as methods of inclusion-exclusion.

- Definition and basic properties of formal power series over the complex numbers.
- Non-labelled combinatorial families and their ordinary generating functions. Interpretation of sum, product, composition, differentiation and integration of ordinary generating functions.
- Labelled combinatorial families and their exponential generating functions. Interpretation of sum, product, composition, differentiation and integration of exponential generating functions.
- Lagrange inversion theorem.

Intended learning outcomes

After passing the course the student should be able to

1. recall the definitions of basic combinatorial notions,
2. apply "the twelve-fold way",
3. apply sieve methods,
4. manipulate formal power series,
5. compute the ordinary generating function of a non-labelled combinatorial family,
6. compute the exponential generating function of a labelled combinatorial family,
7. solve enumerative problems by using the tools in items 2 to 6,
8. communicate with mathematical peers about enumerative combinatorics.

The intended qualitative targets in relation to the Higher Education Ordinance, appendix 2.

Knowledge and understanding

For the Degree of Doctor, the doctoral student shall demonstrate:

- A1: broad knowledge and systematic understanding of the research field as well as advanced and up-to-date specialised knowledge in a limited area of this field.

Teaching formats

Tutorials and seminars.

Examination

INL, written assignment, 3 ECTS credits, concerning learning outcomes 1–7, grade fail (U) or pass (G).

SEM, seminar, 2 ECTS credits, concerning learning outcome 8, grade fail (U) or pass (G).

Grade

Two-grade scale, fail (U) or pass (G).

Requirements

Doctoral student in mathematics/applied mathematics or related field of studies.

Specific entry requirements

Master level in discrete mathematics and algebra.

Selection criteria

1. Doctoral students in mathematics/applied mathematics at Mälardalen University.
2. Doctoral students at Mälardalen University.
3. Doctoral students at other universities in Sweden.
4. Doctoral students at higher education institutions outside Sweden.

Transitional and other provisions

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